Application of Spring Lattice Models to Shell Structures – Geometric and Material Nonlinearities

Report 2017

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1. Introduction

The interest in the numerical analysis of solid continua by spring cell substitutes instead of finite elements and the non-exhaustive state of the art occasioned two complementary doctoral dissertations at the ISD in Stuttgart devoted to the spring cell substitution of continua. The one aims at the exploration of the limits of the approximation of defective bar-spring models [1], the other extends to a rigorous condensed representation of the continuum by introducing additional angular springs [2].

The DAAD project "Einsatz von Stabgittermodellen für Schalentragwerke - Employment of Spring Lattice Models for Shell Structures" is conceived as to investigate the utility of the spring cell models of the ISD in the context of actual structures, and to prove them against the expertise available at Coimbra University in this area. Subject of the first year of the project was the elastic shell [3]. Beyond the shell of the membrane type, the investigations were concerned with the handling of bending stiffness as well.

Based on the first year findings the study of nonlinear issues in the reported second year of the project has been focused on membrane shells. To be specific, the interest has been in the performance of the spring models when applied to problems involving geometrical and material nonlinearities. For this purpose the spring models have been tested against the continuum finite element distinctly in elasticity and in plasticity in the context of large deformations.

2. Task definition and employed tools

The performance of the spring models in the presence of pronounced nonlinearities has been investigated separated in elasticity and in plasticity. This purpose served the two cases designated as "Shape finding" and "Forming" of a membrane shell. In both tests an initially flat circular diaphragm clamped along the periphery is elevated by a homogeneous pressure to assume the shape of a dome. The shape finding is an elastic process, the permanent forming of a shell is by plastic deformation.

Subject of the investigations are the spring models tested previously for their ability to represent linear elastic shells with membrane- and bending stiffness. The *pin-joined bar approach* relies on cells of straight springs. The lattice model employed in the membrane shell problem consists of bar members forming triangular cells. Apart of the common approach referring to the stiffness of the finite element counterpart (the *stiffness approach*), another approach deduced from an approximation of the flexibility (the *flexibility approach*) is seen to represent a favourable alternative [4]. Modelling of the continuum is in general deficient as compared with its finite element counterpart. The introduction of additional angular springs constitutes the *condensed continuum* cell, a complete substitute of the continuum constant strain finite element. Thereby triangular cells of arbitrary shape composed of three straight- and three angular springs are adjusted to the linear-elastic behaviour of an arbitrary material under two-dimensional stress [5]. This is achieved by equating the elastic strain energy of the spring cell and of the continuum at constant strain. The model represents consistently the linear-elastic continuum with all properties condensed in longitudinal and angular springs.

The tasks scheduled for the second year of the project, the treatment of geometric nonlinearities in conjunction with elasticity and plasticity, requires a recourse to the extended theoretical and computational framework of the spring lattice models as outlined in [1] and [2]. Modelling of the plastic continuum by the pin-joined bar cell is found to be sensitive to the appropriate adaption of the material yield stress. The programme implementation of the spring models has been performed individually at the ISD and in Coimbra; the continuum simulations rely on the Finite Element Programming System – FEPS [6] developed in Stuttgart.

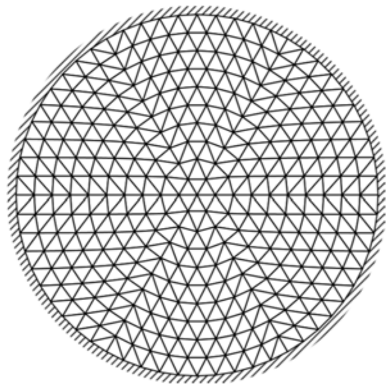

Figure 1: Regular mesh

Two different meshes were employed to analyse the model behaviour with both regular and irregular cell shapes. The generation of triangular grids for arbitrary shaped shells is a demanding task, especially if some kind of regularity is required. Thus methods outlined in [7] were used to generate the regular grid (Figure 1), while the irregular mesh (Figure 2) was generated using gmsh [8].

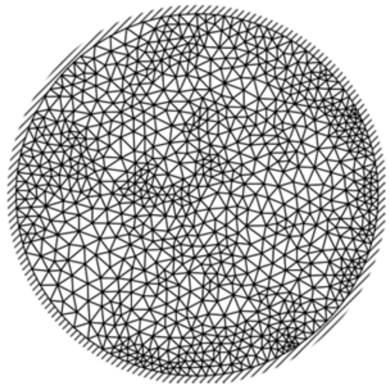

Figure 2: Irregular mesh

3. Elasticity: A form finding process

For the elastic form finding process a constant pressure was applied to the lower surface of the model, and the displacements and strains of the diaphragm were evaluated. A linear relationship between the Green strain and the second Piola-Kirchhoff stress was assumed, with a Young's modulus of 206 GPa and a Poisson number of 0.3 which makes the bar-spring model deficient even if a mesh of equilateral triangles is used, the pertinent number being 1/3 [1,2].

While the displacement results for the condensed continuum (see Figure 3) and the continuum model are (as long as the mesh is sufficiently fine) insensitive to the mesh distortion, only the flexibility approach leads to results which are insensitive to mesh distortion of the bar-spring model (see Figure 4). However, the stiffness approach to the bar-spring model has one remarkable property: It is the only spring model in which the stiffness of the regular, but coarser mesh is smaller than the stiffness of the irregular, but finer mesh. This leads to the conclusion that the mesh regularity has a larger impact on the model stiffness than the mesh cell size.

Overall, the displacement results and, in turn, the stress and strain results for the continuum model, the condensed continuum model and the bar-spring model with the flexibility approach only show minor differences in the results.

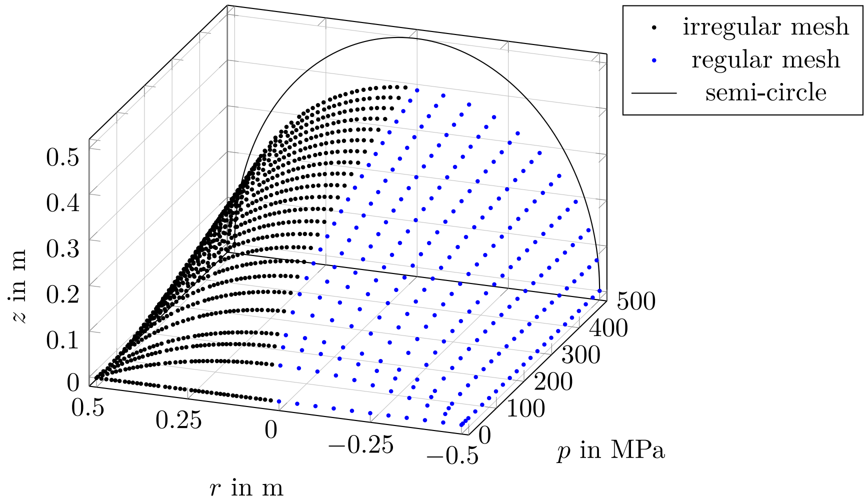

Figure 3: Evolving deformation of the elastic condensed continuum model

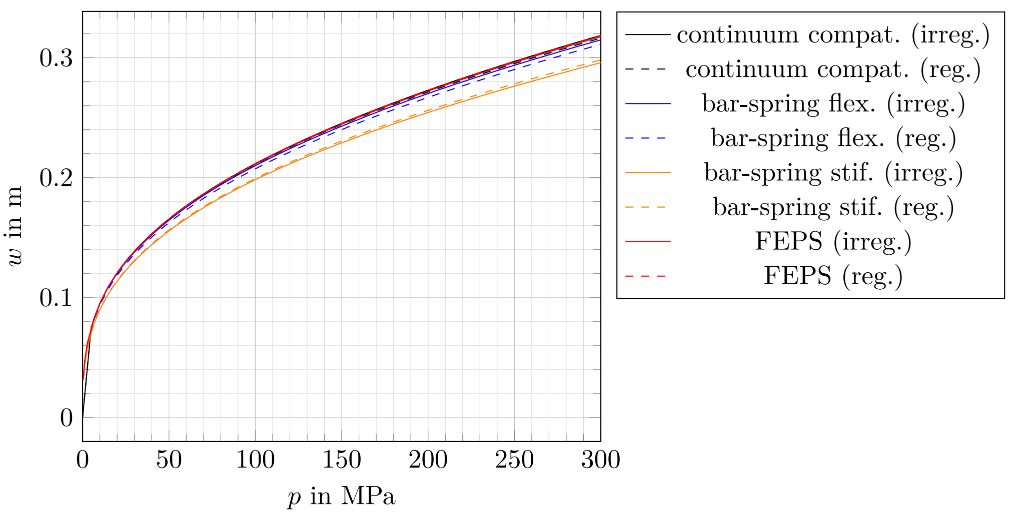

Figure 4: Comparison of the deformation of the different *elastic* models at a given pressure

4. Plasticity: Permanent forming of shell

4.1 Simulation setup

For the simulation of the form giving process, the previously used elastic material is extended using a von Mises (J_2) plasticity model. The yield function is defined in terms of the Cauchy-stress, and the plastic strain in terms of natural (logarithmic) strain. Linear strain hardening was assumed: $s_c = s_{c,0}\left(1 + \alpha \int_{e_p} de_p\right)$, where s_c is the current yield stress, $s_{c,0} = 235$ MPa is the initial yield stress, $\alpha = 2$ is the linear hardening parameter and e_p is the equivalent plastic strain. A graphical representation of this material law is shown in Figure 5.

As in the elastic case, the model is subjected to a constant pressure on the lower surface.

All results are shown for the deformed diaphragm subjected to load. The unloaded state is not shown separately, as its shape is almost indistinguishable from the loaded shape.

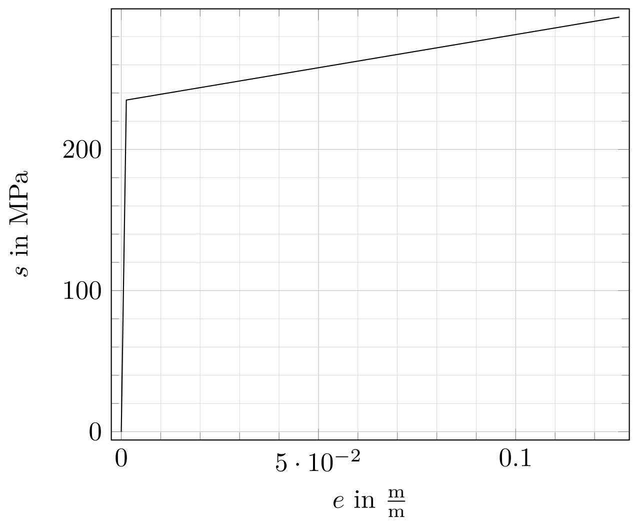

Figure 5: Uniaxial Stress strain relationship of the material

4.2 Simulation results

The load-displacement-diagrams for the elasto-plastic simulations (Figure 7) show a similar behaviour to the elastic results: The differences between the regular and the irregular mesh are almost indistinguishable apart from the bar-spring model with the stiffness approach. The only exception is near the limit point of the system, where the remaining stiffness of all models is strongly influenced by the mesh.

The differences of the different models in the elasto-plastic simulation are small, but visible during the whole plastic regime. This is an expected result, as the implementation of the given plasticity model differs between the discretization techniques (e.g. the bar-spring model uses a one-dimensional approximation). As a result of these differences, the limit point as well as the limit behaviour of the models is different: the results for the limit load differ by less than 25%.

Another note of interest is the different convergence behaviour of the continuum model and the continuum-compatible model while the load step size is reduced, as can be observed in Figure 8. As the latter model is implemented in strain space, a semi-analytical solution for the incremental plastic strain can be used, thus making the model insensitive to the size of the strain increment[1]. As a result, the load-displacement-curves in Figure 8 are indistinguishable, whereas the continuum-model (FEPS) shows a strong dependence on the load steps near the limit state. It is noted that both cases apply the radial return technique; the continuum model in the raw fashion, the other refined with the above mentioned semi-analytic technique.

[1] That is, as long as the strain increment is sufficiently small, which can be achieved through sub-incrementation of the material law.

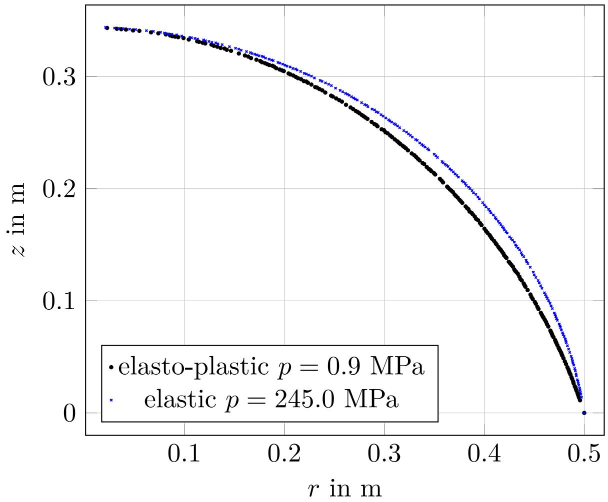

Figure 6: Comparison of the elastic and elasto-plastic dome shapes for similar elevations

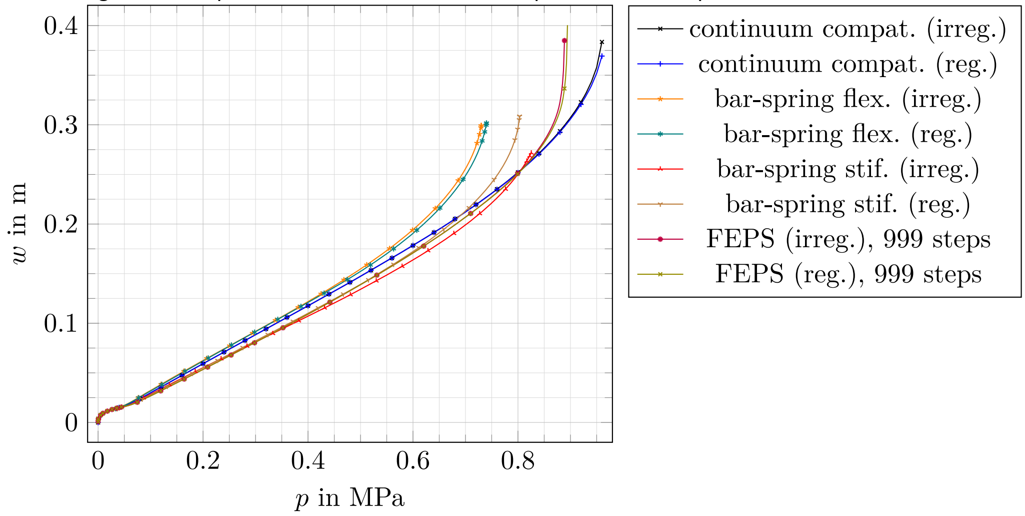

Figure 7: Displacement-load-curves at the centre-point of the diaphragm.

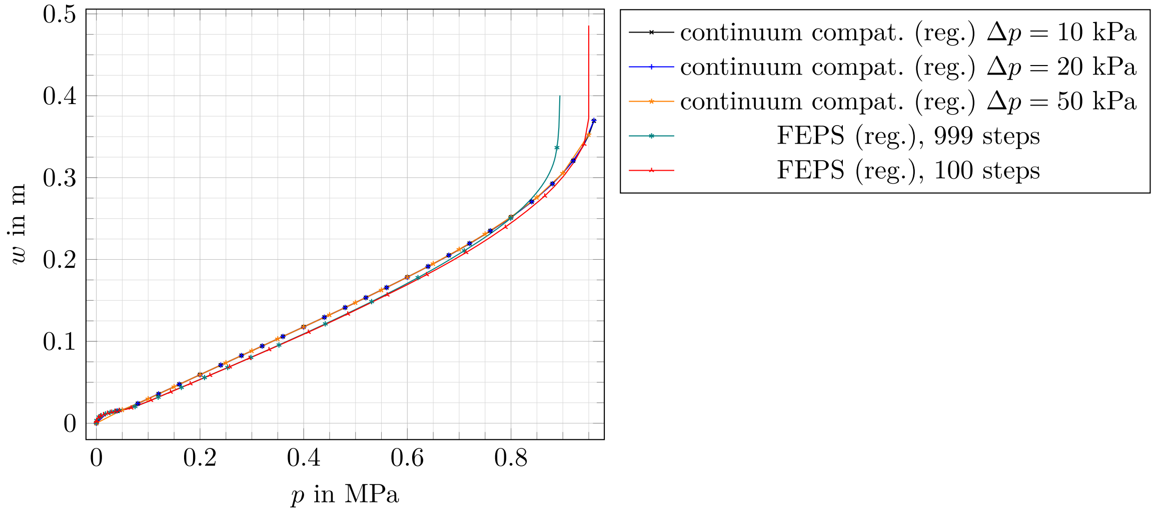

Figure 8: Displacement load curves: Change due to number of load increments

5. Concluding remarks

Following the investigation of the linear spring models in the first phase of the project the work in the second phase was devoted to nonlinear shell problems. For this purpose the theoretical background of the developed spring models has been elaborated for the treatment of geometric nonlinearities of membrane shells in conjunction with elastic as well as with plastic behaviour of the material. The test cases selected for this purpose have been the elastic “Form finding” of a dome by blowing a flat circular diaphragm, on the one hand, and its permanent “Forming” to a membrane shell by plastic deformation, on the other hand.

The elastic nonlinear computations confirm the equivalence of the generalized spring cell (straight and angular springs) with the continuum finite element counterpart, and the adequacy of the simpler straight spring cell for the analysis of the nonlinear shell problem. On this occasion, the stiffness- and the flexibility approach were distinguished, their performance quantified to the favour of the flexibility concept. The investigation of the elastic-plastic variant of the geometrically nonlinear problem added substantial knowledge with regard to the application of the spring cell models. The deformation behaviour of the finite element model is reproduced to a satisfactory degree of accuracy. Even the limit state can be reproduced but with an error in the range of 25% of the applied load. The implication of plasticity in connection with the spring models has postponed the damage issue, initially scheduled for this second phase, beyond the time frame of the present project of scientific exchange. The task is to be accomplished, however, in the continuing collaboration of the project partners by an implementation of a softening material model, [9], in the spring cell models.

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Das zweijährige Projekt „Einsatz von Stabgittermodellen für Schalentragwerke“ wurde gefördert vom DAAD aus Mitteln des Bundesministeriums für Bildung und Forschung – BMBF.