Application of Lattice Spring Models for Shell Structures

Report 2016 – The Elastic Shell

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Introduction

A number of arguments may suggest that solid continua could be investigated with spring cell substitutes instead of finite elements. The main arguments would be a simplified discrete representation, microstructural modelling, uniaxial material laws, the resolution of progressing damage and failure. However, the facts damp the high expectations. Only specific continua can be represented by spring cells.

There is considerable contemporary research being done on Lattice Spring Models. The initiative work has been performed by Hrennikoff [1]. In contrast to the representation of solid continua by spring grids also grid structures were analysed by continuum methods [2]. Given the non-exhaustive state of the art, two complementary doctoral dissertations at the ISD in Stuttgart are devoted to the spring cell substitution of continua. One aims at defining the limits of the approximation of defective bar-spring models, the other extends to a rigorously condensed representation of the continuum by introducing additional angular springs.

An advanced stage of the research work [3] [4] was exposed and discussed at the international ECCOMAS 2016 Congress in the framework of the thematic session “Spring Lattice Models for Linear- and Nonlinear Continua”. The report on the work performed at Coimbra University on the grid generation and stress analysis of shell structures [5] completed the available background as the starting point of the scheduled DAAD project. Also of interest was the diverse approach to spring cells based on the natural representation of the elastic continuum [6].

The project “Einsatz von Stabgittermodellen für Schalentragwerke - Employment of Spring Lattice Models for Shell Structures” was conceived for investigating the utility of the ISD spring cell models in the context of shell structures, and for proving them against the expertise available at Coimbra University. The first of the two year project deals with the elastic shells, the subject of the present account. Apart from membrane shells, the investigations also deal with the modelling of shell structures with bending stiffness. A specific programme implementation of the spring models is used at the ISD, whereas Coimbra relies on the Finite Element Programming System – FEPS [7] developed in Stuttgart.

A Preliminaries

A.1 Contrasting Continuum- with Reticulated Shells

Arches and shells can balance external forces without significant bending moments, provided that they have the adequate anti-funicular shape of the external forces. Arches and single-curvature shells show the same structural behaviour characterized by a very low tolerance to loading changes, in the sense that even small non-proportional loading changes require significant bending moments to balance the external forces. Double curvature shells have a more robust behaviour in this respect, since membrane forces remain capable to balance external loading even in presence of significant non-proportional loading changes.

Grid shells display an arch-like behaviour, if they are not triangulated, irrespective of single or double curvature, while double curvature triangulated grid shells behave like double curvature continuum shells.

A.2 On the Analysis of Reticulated Shells

The computational capacity of computers in the nineteen-sixties was not sufficient for the numerical treatment of space frame shells. Such wide-span structures could comprise nearly half a million discrete members connected at several thousands of joints. Therefore a recourse to solutions for the continuum should help assessing the resistance of the space frame against the applied forces.

The continuum approach implies the establishment of an equivalence between space frame and continuum shell. Given the space frame design, elastic properties and effective thickness of the continuum counterpart are determined analogously. Such a methodology is found in [8] for single layer, reticulated, shells composed of triangular assemblies of straight bar members. Thereby, member forces are related to membrane forces, the elastic constants and effective thickness of the continuum shell are specified and stability is discussed regarding the buckling of individual members, local snapping through and global shell failure. As a particular result it is concluded that the continuum analogue to the skeletal structure should be a shell with larger thickness but low elastic properties. Accordingly, the space frame functions as a thick shell of rather spongy material. In [9] the approach is extended to the continuum analysis of double layer space frame shells on the basis of tetrahedral assemblies of elastic bar members.

Structural and theoretical aspects of these remote studies appear nowadays actual in connection with the present subject of interest: the investigation of continua through the use of equivalent frameworks. Among others, a design principle of reticulated shells might be kept in mind when generating space grids to model continuum shells: to minimize the variety of members and connections required.

B Theoretical Background

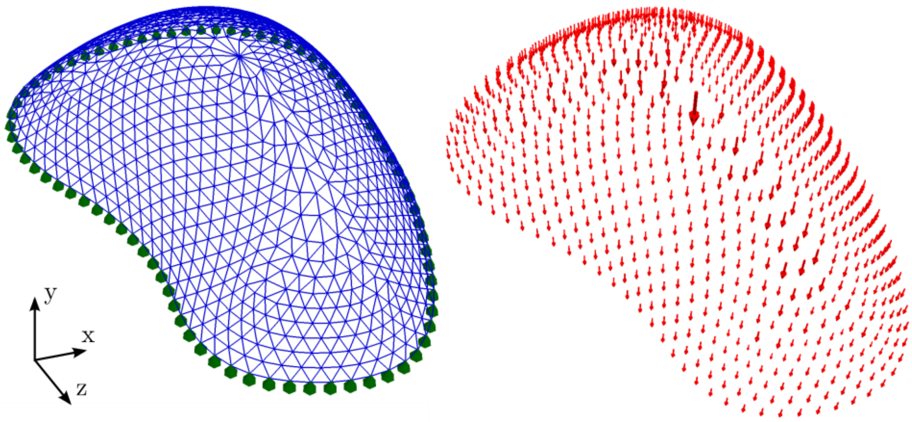

Figure 1. Shell No 1 with boundary displacement boundary conditions, which are represented by cones (left) and the load condition (right).

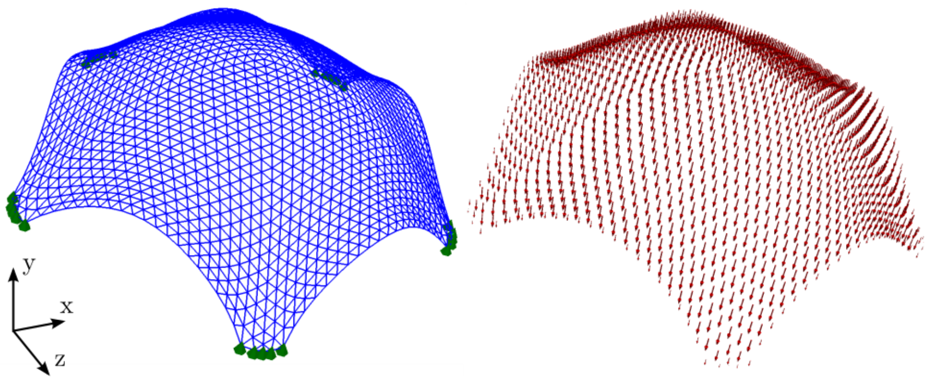

Figure 2. Shell No 2 with boundary displacement boundary conditions, which are represented by cones (left) and the load condition (right).

B.1 The Beam Lattice Model

Beam lattice models are adequate to represent continuum shells, since they model the bending behaviour, while the stress state is much simpler, since it is one-dimensional, provided that shear stresses in the shell can be neglected (which is usually the case). Features like instability behaviour are qualitatively similar.

In relation to pin-joined lattices, their ability to model continuum shells has advantages in case of low curvatures, since it is avoided that the tangent stiffness matrix becomes singular, while keeping the stress distribution in the bar's cross-section nearly homogeneous. This happens as long as the bending moments are not needed to balance the external forces, as is the case for triangulated lattices. On the other hand, pin-joined models are simpler to implement, especially in the case of large displacements, since no rotational degrees of freedom need to be taken into consideration. Besides, their yielding and damage behaviour is easier to define.

B.2 The Pin-joined Bar Approach

A lattice spring model with normal springs can be used to model two- and three-dimensional continua. The material parameters are restricted by the Cauchy relations, so that only materials with certain properties can be employed. The stiffness of the trusses can be determined by comparing the strain energy of the lattice and the continuum, which only match in certain cases e.g. equilateral triangles and rectangles. If distorted cells are used, the stiffness of the lattice can be approximated through two conditions. First it is assumed that the strain energy of the lattice equals the strain energy of the continuum for a constant strain state. Secondly it is assumed that the stiffness is homogeneously distributed over the lattice for an isotropic material. This method results in an optimized stiffness model, whereby the discretization error between the lattice and the continuum is minimized. This approach is also applicable on shells.

B.3 The Condensed Continuum Model

As presented in a recent paper [10], one can use triangular cells of arbitrary shape comprising three normal springs and three angular springs to represent the linear-elastic behaviour of an arbitrary material for two-dimensional mechanical systems. The spring constants can be obtained by assuming equal elastic strain energy of the lattice model and the continuum model under a constant strain state.

Though this model is derived for two-dimensional problems, the transition to the three-dimensional space along with the introduction of a finite thickness and the assumption of a plane stress state in the elementary cell directly yields a model for membranes in three dimensions.

The model consistently represents the elastic continuum in constant strain elements with all properties condensed in longitudinal and angular springs.

B.4 Generation of Triangular Grids for Shell Computation

Generating triangular grids for arbitrarily shaped shells is a demanding task, especially if some kind of regularity is required. Usual approaches include the use of the Delaunay criterion, advancing fronts, either from one side or in a ring-wise construction from the outer contour, etc. . Aspects of the grid generation technique were presented in [5].

C Numerical Investigations

Numerical investigations have been performed to test the viability of modelling shells by means of single layered pin-joined bars and beam lattice models. The first ones exhibit numerical difficulties in the cases of low curvatures. Beam lattice models show a qualitatively similar behaviour as continuum shells. Models of double layered pin-joined lattices have been developed and are in the initial stage of testing.

C.1 Membrane Shell

For the comparison of the condensed continuun model with the defective stiffness model, a linearized deformation analysis of shells under a constant gravity load is carried out. These calculations yield two main results. Firstly, as expected, solutions can only be obtained for configurations with a double curvature over the whole shell. Areas lacking this double curvature have no stiffness perpendicular to the surface.

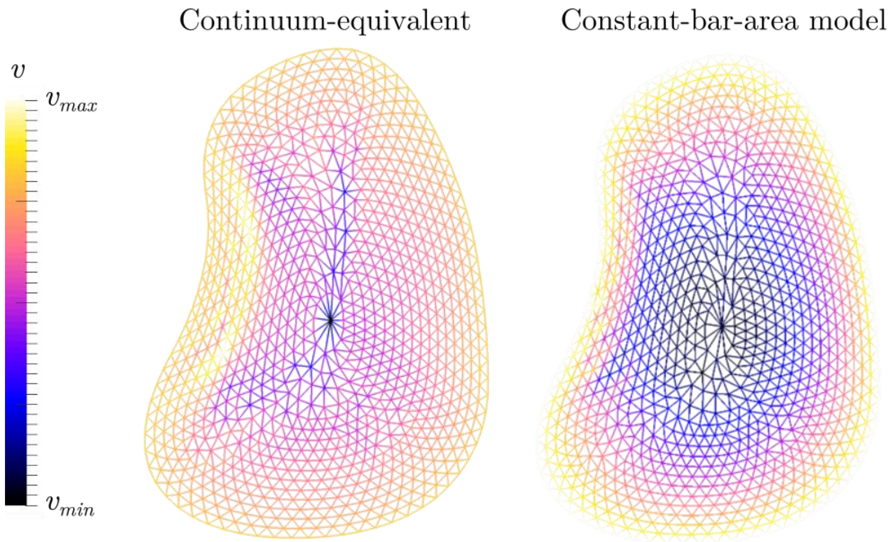

Figure 3: Vertical displacement of the different models. The absolute values of the displacements

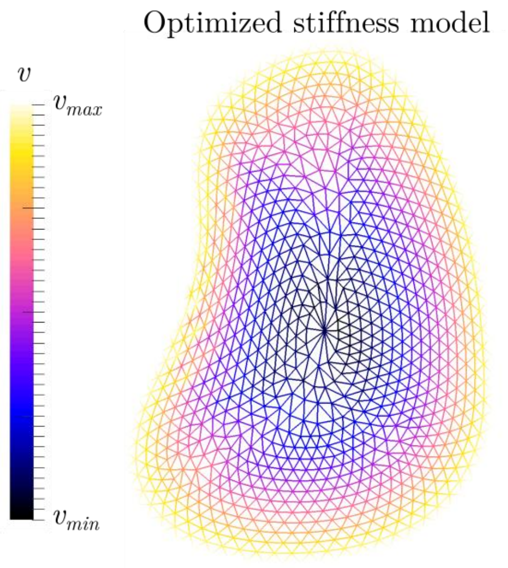

Figure 4: Vertical Displacement of optimized stiffness model. The absolute values of the displacements

Secondly, the displacement results for shell No 1 with the consistent, condensed continuum and the optimized stiffness model Figure 3 and Figure 4 are much less smooth (folds) than the results for both the beam model and pin-joined bar model with a constant bar area, see e.g. Figure 5. On the one hand it was found that these results do not depend on the form-finding procedure for the unloaded shell, if it is based either on a membrane or on a pin-joined bar model. On the other hand it can be shown that the folds are caused by the geometry of the Shell

No. 1 at the regions of low curvature. A model with bending stiffness (double-layer model, Figure 6) results to a smooth displacement field without appreciable folds.

It is suspected that the results of the consistent, condensed continuum model may be improved by including additional shell-bending related angular springs, which restrict the out-of-plane angle changes of adjacent triangular cells. This should be of concern in future investigations.

A similar result to the condensed continuum model is obtained for shell No 1 by the optimized stiffness lattice model as shown in Figure 4.

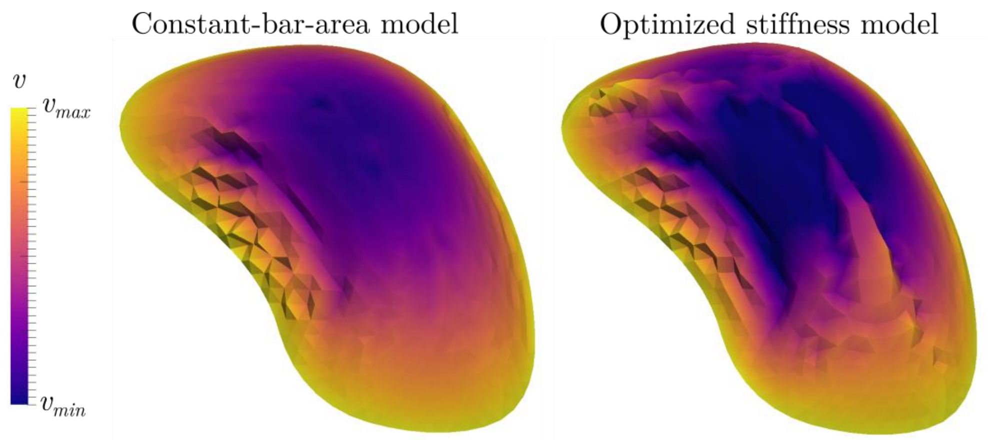

Figure 5: Deformed constant-bar-area and optimized stiffness model of Shell No. 1

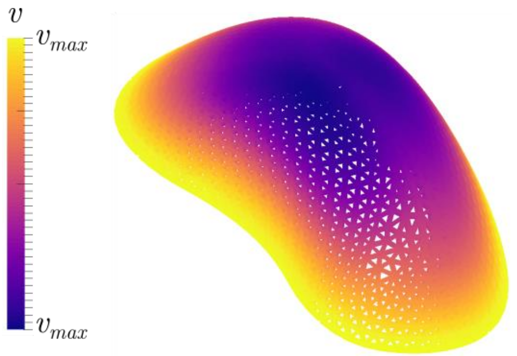

Figure 6: Displacement v of a double-layer model of Shell No. 1

C.2 Bending Stiffness

Shells with a low curvature require a bending stiffness, otherwise instabilities can occur due to missing out-of-plane stiffness. Shell No 2 shows such instabilities and therefore cannot be computed with a single layer normal spring model. In a linear simulation rigid body motions occur at unstable areas. This areas can be located by means of curvature calculations as an indicator. To evaluate the shell curvature, the angle φ between the normal vectors of adjacent nodes can be determined. The minimum angle of each node is depicted in Figure 7 and points out the areas with a low curvature. Figure 7 shows that the transition between the gates and pedestals is the critical area.

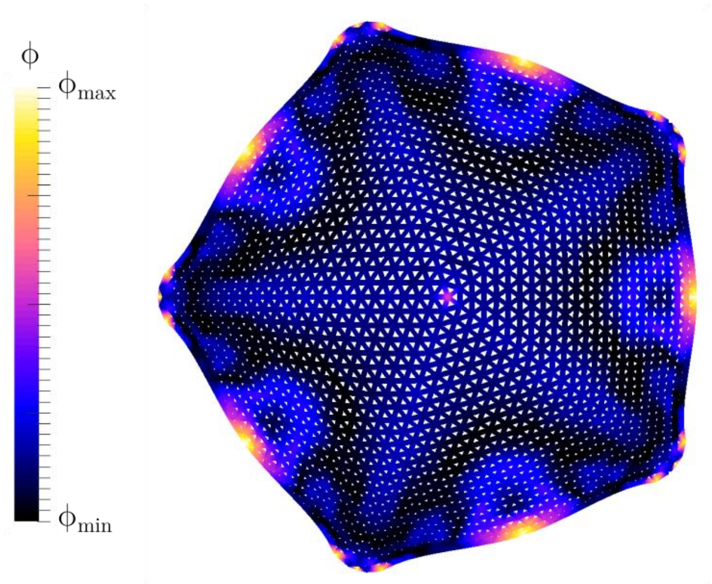

Figure 7. Minimal angle φ between the normal vectors of adjacent nodes

One option to avoid this restriction of the single layer model is the extension to a volume model. The volume model is generated by blowing out the surface in both directions. The lattice is equipped with wedge cells (Figure 8). The result for the vertical displacement is shown in Figure 9. Another option is the introduction of a nonlinear geometric model with pre-stress, which could be part of prospective studies.

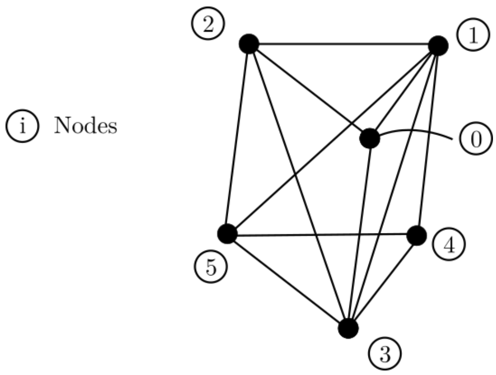

Figure 8. Cell of the volume model consisting of three tetrahedrons.

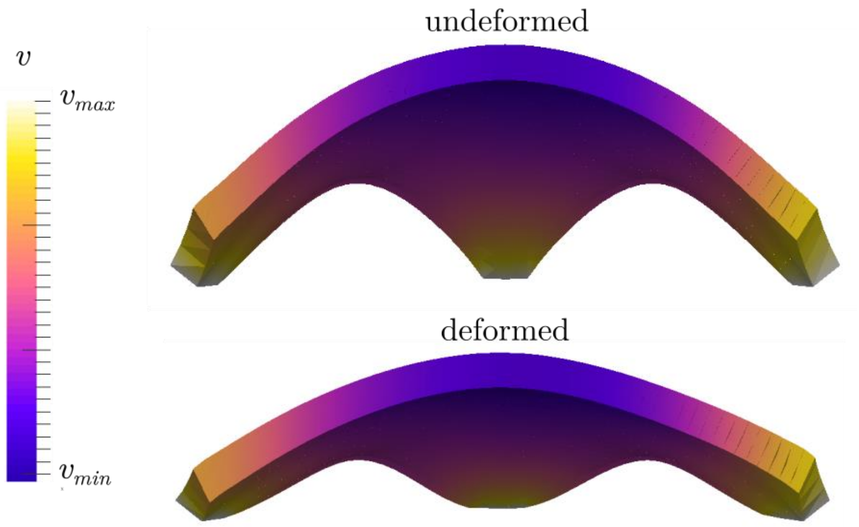

Figure 9. Vertical displacement of the half symmetric-shell model. The absolute displacement depends on the thickness of the shell.

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